

Broadband Slow Light Metamaterial Based on a Double-Continuum Fano Resonance

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We propose a concept of a low-symmetry three-dimensional metamaterial exhibiting a Double-Continuum Fano (DCF) optical resonance. Such metamaterial is described as a birefringent medium supporting a discrete “dark” electromagnetic state weakly coupled to the continua of two nondegenerate “bright” bands of orthogonal polarizations. It is demonstrated that light propagation through such DCF metamaterial can be slowed down over a broad frequency range when the medium parameters (e.g. frequency of the “dark” mode) are adiabatically changed along the optical path. Using a specific metamaterial implementation, we demonstrate that the DCF approach to slow light (SL) is superior to that of the EIT because it enables spectrally uniform group velocity and transmission coefficient.

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The ability to slow down light to low group velocities v_g compared with the vacuum light speed c while maintaining high coupling efficiency [1] is one of the most dramatic manifestations of controlled light manipulation in optics. Apart from its fundamental significance, it has long-reaching technological applications [2], including enhanced nonlinear effects due to the energy density compression by as much as c/v_g ; pulse delay and storage for optical information processing [3]; optical switching, and quantum optics. Most approaches to obtaining SL rely on the phenomenon of Electromagnetically Induced Transparency (EIT) [4]. EIT and its analogs have been demonstrated in several media, including cold [1], warm atomic gases [5], and even plasmas [6, 7].

More recently, in response to the emerging applications such as bio-sensing, an increasing attention has shifted towards obtaining EIT using electromagnetic metamaterials [8–10]. Metamaterials enable engineering electromagnetic resonances with almost arbitrary frequencies and spatial symmetries. For example, the EIT phenomenon has been emulated in metamaterials [8–10] possessing two types of resonances: a “dark” one, which is not directly coupled to the incident electromagnetic field, and a “bright” one, which is strongly coupled to the incident field. If the respective frequencies of these resonances, ω_Q and ω_R , are very close to each other, they can become strongly coupled by a slight break of the metamaterial’s symmetry.

The most serious limitation of the EIT-based SL (EIT-SL) stems from the desirability of achieving the SL over a broad spectral range, especially for optical buffering of ultra-short pulses. Increasing coupling between the bright and dark resonances broadens the spectral range, yet comes at the expense of increasing both the group velocity v_g and group-velocity dispersion $d(v_g^{-1})/d\omega$. Qualitatively, the bandwidth limitation of the EIT-SL arises because the flatness of the EIT transmission band (which is necessary for small $d\omega/dk_z$, where ω and k_z are, respectively, the frequency and the wavenumber of the propa-

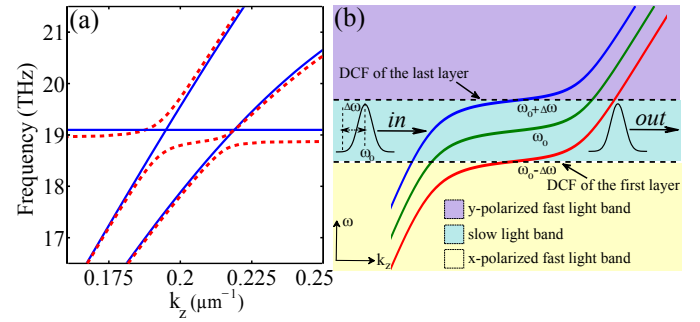


FIG. 1: (Color online) (a) PBS of DCF metamaterial as predicted by the analytical model of coupled oscillators [see Eqs. (1, 2)]. Different line styles correspond to the reduction of the spatial symmetry of a metamaterial: from two mirror-symmetry planes (solid) to none (dashed). (b) Broadband SL: the medium is comprised of multiple layers of DCF metamaterials with spatially-varying resonance frequency Ω_Q in the $\omega_0 - \Delta\omega < \Omega_Q < \omega_0 + \Delta\omega$ range. The incoming light undergoes polarization transformation and is slowed down.

gating radiation) originates from the finite spectral width $2\Delta\omega$ of the EIT band. Therefore, only the light inside the $\omega_0 - \Delta\omega < \omega < \omega_0 + \Delta\omega$ bandwidth can be slowed down. Using metamaterials with spatially-dependent $\omega_0(z)$ along the propagation direction z cannot increase the total propagation bandwidth of the EIT-SL because the EIT band is surrounded by the stop bands as illustrated below. Therefore, a natural solution to the bandwidth problem is to design a metamaterial supporting a propagating mode which is *spectrally broad, yet possesses a “flat” segment* with $\partial\omega/\partial k_z \ll c$, as is schematically shown in Fig. 1.

In this Letter we propose a new approach to producing SL which relies on the phenomenon of Double-Continuum Fano (DCF) [11] optical resonance: coupling of a single discrete state (“dark” mode) to the two sets of continuum states (two propagating modes of different polarization states). If the frequency of the “dark” mode is

embedded in the frequency continua of the propagating modes, then a simple symmetry breaking couples all three modes. The result of such coupling is a very unusual propagation band [see Fig. 1(a)] which fulfills the above requirement for broadband SL applications by continuing from one propagating mode to another, with the SL region in between. We demonstrate that multiple layers of such DCF metamaterial with adiabatically changing frequency of the “dark” mode [as schematically explained in Fig. 1(b)] give rise to broadband SL with spectrally uniform group velocity and transmission. Unlike the more commonly known single-continuum Fano resonance [12, 13] that can be observed with highly symmetric molecules possessing at least one reflection mirror symmetry, DCF resonance requires low spatial symmetry molecules that are not reflection-symmetric with respect to any plane passing through the direction of light propagation.

Before introducing a specific metamaterial realization, we consider a very general case of a DCF medium comprised of anisotropic molecules containing two “bright” transitions (characterized by the oscillator strengths q_x and q_y , respectively, coupled to the two light polarizations) and one “dark” transition (characterized by the oscillator strength Q and decoupled from the light). The equations for the coupled oscillator strengths q_x , q_y , and Q excited by the light with the electric field components E_x and E_y are:

$$\begin{aligned}\ddot{q}_x + \Omega_{Rx}^2 q_x + \kappa_{xy} q_y + \kappa_{xQ} Q &= a\alpha_x E_x e^{i\omega t}, \\ \ddot{q}_y + \Omega_{Ry}^2 q_y + \kappa_{xy} q_x + \kappa_{yQ} Q &= a\alpha_y E_y e^{i\omega t}, \\ \ddot{Q} + \Omega_Q^2 Q + \kappa_{xQ} q_x + \kappa_{yQ} q_y &= 0,\end{aligned}\tag{1}$$

where $\Omega_{Rx/Ry/Q}$ are the resonant frequencies of the transitions and $\kappa_{xy/xQ/yQ}$ are the coupling coefficients between them. q_x and q_y are coupled to external fields through the coupling constants $a\alpha_x$ and $a\alpha_y$, where a is a constant with a dimension of ω^2 , and $\alpha_{x/y}$ are the depolarization factors such that the dipole moment *normalized to one molecule’s volume* V_0 is $p_{x/y} = \alpha_{x/y} q_{x/y}$. On the other hand, the dark state Q cannot be directly excited by the electric field, nor does it directly contribute to the polarizability of the medium, thereby playing the role of the discrete Fano state.

Solving for q_x and q_y in the form of $q_i = b_{ij} E_j$ (where $i=x,y$), we can construct the dielectric permittivity tensor $\hat{\epsilon} = \hat{1} + 4\pi\hat{\chi}$, where $\hat{\chi} = (N_0 V_0) \hat{a} \hat{b}$, and $\hat{a} = \text{diag}[\alpha_x, \alpha_y]$, where N_0 is the molecular density. The effective polarizability of the DCF medium is given by:

$$\begin{aligned}\chi_{xx} &= N_0 a \alpha_x^2 (W_{Ry} W_Q - \kappa_{yQ}^2) / D \\ \chi_{yy} &= N_0 a \alpha_y^2 (W_{Rx} W_Q - \kappa_{xQ}^2) / D \\ \chi_{xy} &= \chi_{yx} = -N_0 a \alpha_x \alpha_y (\kappa_{xy} W_Q - \kappa_{xQ} \kappa_{yQ}) / D\end{aligned}\tag{2}$$

where $D = W_{Rx} W_{Ry} W_Q - \kappa_{xQ}^2 W_{Ry} - \kappa_{yQ}^2 W_{Rx} - \kappa_{xy}^2 W_Q - 2\kappa_{xQ} \kappa_{yQ} \kappa_{xy}$ is the determinant of the matrix formed by the l.h.s of Eq. (1), and $W_i = (\Omega_i^2 - \omega^2)$. Photonic band structure (PBS) is analytically calculated from the eigenvalue equation for $k_z(\omega)$: $\det(k_z^2 \delta_{ij} - \epsilon_{ij} \omega^2 / c^2) = 0$, which directly follows from the Maxwell’s equations.

Unlike the EIT case where the dark mode’s frequency needs to be matched to that of the x -dipole resonance [8, 10], the DCF occurs under the following conditions: (i) $\Omega_Q < \Omega_{Rx}, \Omega_{Ry}$, and (ii) at least 2 of the 3 coupling constants (κ_{xQ} , κ_{yQ} , and κ_{xy}) are non-vanishing. Without significant loss of generality, and with an eye on the specific metamaterial implementation (see schematic in Fig. 2), we have neglected the direct coupling between q_x and Q . Finite values of the κ ’s is the consequence of the reduced spatial symmetry of the anisotropic molecule. For example, $\kappa_{xy} = 0$ would be either accidental, or due to the high spatial symmetry of the molecule (e.g., $y-z$ mirror symmetry). Figure 1(a) shows the PBS in the DCF medium (dashed line) calculated from the analytical model using the following parameters: $[\Omega_{Rx}, \Omega_{Ry}, \Omega_Q] = [27, 23.85, 19.1]$ THz, $[\kappa_{xQ}, \kappa_{yQ}, \kappa_{xy}] = [50, 0, 50]$ THz², $\alpha_x = 1$, $\alpha_y = 1.65$, $a = 529$ THz², and $N_0 V_0 = 10^{-3}$.

The unusual PBS in the DCF medium can be understood through its evolution from that in the medium comprised of the symmetric molecules ($\kappa_{xQ} = 0$, $\kappa_{xy} = 0$) [solid lines in Fig. 1(a)]. First, the $\kappa_{xQ} \neq 0$ coupling hybridizes the x -dipole and the quadrupole resonances and creates an avoided crossing between the x -polarized propagation band and the discrete “dark” mode. The second avoided crossing between the y -polarized propagation band and the discrete “dark” mode occurs owing to $\kappa_{xy} \neq 0$.

Thus, the DCF medium comprised of the low-symmetry molecules supports a propagation band that smoothly connects the two orthogonally polarized continua by passing through the SL region near the “dark” mode’s frequency Ω_Q . Such PBS has the following advantages over that in the EIT medium: (i) the dark mode frequency has more tunability because it is only required to be embedded inside the continuous bands, and (ii) there are no bandgaps on either side of the SL region. These features of the SL band are necessary for realizing the broadband SL using the approach of adiabatically-varying material parameters [specifically, $\Omega_Q(z)$] schematically shown in Fig. 1(b).

Both DCF and EIT metamaterials supporting the propagation of slow light can be implemented using a three-dimensional periodic arrangement of the unit cells comprised of three metallic antennas. Such unit cell shown in Fig. 2 consists of 2 vertical and 1 horizontal metallic antennas embedded in a dielectric medium with $n = 1.5$. It is reminiscent of the ones used for single-layer EIT metamaterials [8, 10], except that both s_x and s_y are allowed to be non-vanishing, thereby dispensing with all reflection symmetries of the unit cell. The two vertical

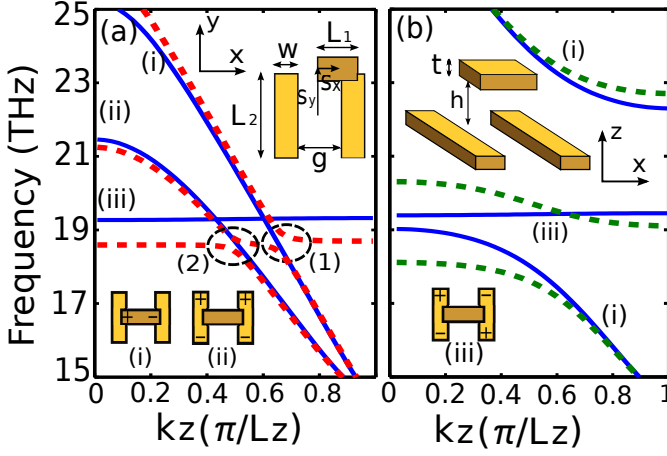


FIG. 2: (Color online) PBS for SL metamaterials based on the DCF resonance (a) and on the EIT (b). The insets show the geometry and dimensions of the unit cell and the three supported resonances: (i) horizontal dipole, (ii) vertical dipole, and (iii) the quadrupole. Solid lines: PBS computed for a symmetric unit cell ($s_x = 0$ and $s_y = 0$). (a) Propagation bands for DCF-based metamaterial (dashed lines): $s_x = 700$ nm, $s_y = 2$ μ m, $L_1 = 2$ μ m. The avoided crossings marked (1) and (2) are caused by $s_y \neq 0$ and $s_x \neq 0$, respectively. Flat portion of the spectrally-extended middle band: SL. (b) Propagation bands for the EIT-based metamaterials (dashed lines) with partial symmetry breaking ($s_x = 0$ nm, $s_y = 500$ nm, $L_1 = 3.8$ μ m): emergence of SL for the spectrally-narrow middle band. For (a) and (b): $t = 400$ nm, $h = 400$ nm, $L_2 = 4$ μ m, $w = 800$ nm, $g = 2.2$ μ m. The metamaterial's periodicities are 6 μ m \times 7 μ m \times 7 μ m, and the electromagnetic waves are assumed to propagate in the z direction.

antennas support a bright dipole and a dark quadrupole resonances, while the single horizontal antenna supports another bright resonance. Within the general model described by Eqs. (1), the strengths of these resonances are measured by q_y , Q , and q_x , respectively. Coupling between all 3 resonances is induced by breaking the reflection symmetries of the structure. For example, $s_y \neq 0$ results in $\kappa_x Q \neq 0$, while $s_x \neq 0$ results in $\kappa_{xy} \neq 0$.

The PBS, calculated using the finite-elements software COMSOL, is shown in Fig. 2 for different geometric parameters of the unit cell. For example, by approximately matching Ω_{Rx} and Ω_Q (this is done by choosing the appropriate length L_1 of the horizontal antenna) and selecting $s_x = 0$, $s_y \neq 0$, an EIT-SL band is obtained as indicated by the middle dashed line in Fig. 2(b) (y -polarized bands are not shown). Note that the EIT-SL band is surrounded by the two stop-bands. On the other hand, when (i) $s_x \neq 0$, and (ii) the dark mode of the fully-symmetric structure ($s_x = s_y = 0$) intersects the two propagation continua (solid lines), a DCF-SL band emerges as shown in Fig. 2(a) (dashed lines). The difference between the DCF bands in Figs. 1(a) and 2(a) is

caused by the band-folding owing to the periodic nature of the metamaterial in the z -direction.

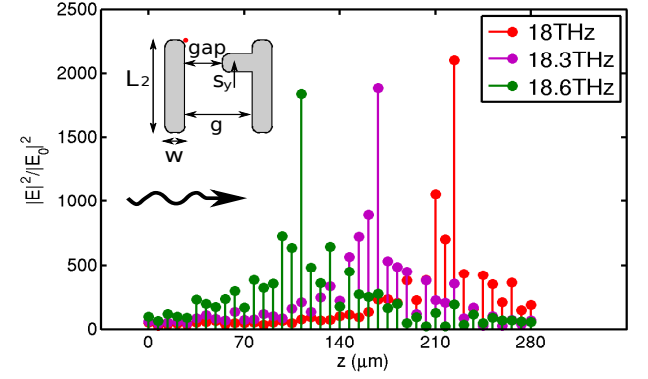


FIG. 3: (Color online) Field enhancements of four different frequency components propagating in an adiabatically varying DCF-based metamaterial. Inset: single-layer unit cell. Spectral position of the SL is adiabatically varied: $L_2[\mu\text{m}] = 3.75 + z/700$. Field intensity is calculated at the red spot shown in the inset. Other parameters: $s_y = L_2/4$, $w = t = 0.8$ μ m, $g = 2.7$ μ m, and $\text{gap} = 1.7$ μ m.

While it is apparent from the PBS that light can be significantly delayed by either EIT or DCF-based metamaterial, the spectral width of such SL band is quite limited. To create a broadband SL, we propose a multi-layered structure with each layer having its DCF's resonance frequency $\approx \Omega_Q$ adiabatically varied along the propagation direction z . In the metamaterial structure shown in Fig. 2 such tuning can be achieved, for example, by changing the length of the 2 parallel antennas. Because the SL mode in the DCF-based metamaterial contains no band gaps, the adiabatic process smoothly converts one fast mode (e.g., x -polarized) into the hybrid slow mode, and then into another fast mode (y -polarized) without significant reflection. Such conversion is not possible for EIT-SL because the SL band is surrounded by the stop bands which lead to reflection of the SL as was recently demonstrated [14, 15].

Specifically, we consider a broadband x -polarized laser pulse with the central frequency ω_0 and bandwidth $\Delta\omega$ incident upon an N -layer metamaterial with adiabatically varying $\Omega_Q^{(1)} < \Omega_Q^{(j)} < \Omega_Q^{(N)}$ (where j is the metamaterial's layer number) as shown in Fig. 1(b). It is further assumed that, while $\Delta\omega$ is much larger than the spectral width of the SL portion of any individual layer, the following relations are satisfied: $\Omega_Q^{(1)} < \omega_0 - \Delta\omega$ and $\omega_0 + \Delta\omega < \Omega_Q^{(N)}$. As the pulse passes through the structure, each frequency component ω is slowed down inside its corresponding layer j satisfying $\Omega_Q^{(j)} \approx \omega$ while propagating with no significant delay through the other layers. The light pulse is also gradually converted from x - into y -polarized. Because all the frequency components of the pulse undergo the same adiabatic transition, the entire

pulse is slowed down uniformly with no group velocity dispersion.

COMSOL simulations of a $N = 41$ -layer DCF-SL metamaterial, with the single-layer unit cell shown in the inset to Fig. 3, were performed. Note that this unit cell, while topologically equivalent to the one shown in Fig. 2, is optimized to decrease Ohmic losses. Antennas are assumed to be made of silver with the dielectric permittivity described by the Drude model [16]. The structure is triply-periodic with the period $L = 7 \mu\text{m}$ and embedded in a dielectric with $n = 1.5$. The SL's frequency is adiabatically varied from layer to layer by increasing L_2 from $3.75 \mu\text{m}$ to $4.15 \mu\text{m}$. Other unit cell parameters are given in the caption. Field intensity enhancements evaluated at the corner of the left vertical antenna are shown in Fig. 3 for 3 different frequencies. As Fig. 3 shows, different frequencies are slowed down inside different layers as indicating the corresponding field enhancement. Thus, light is slowed down over the entire spectral band $\Omega_Q^{(1)} < \omega < \Omega_Q^{(N)}$.

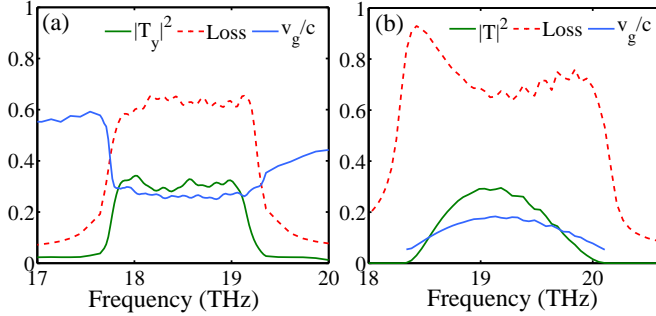


FIG. 4: (Color online) Transmission (solid green lines), absorption (dashed red lines), and group velocity (solid blue lines) of (a) adiabatic DCF-based and (b) EIT-based metamaterials. x -polarized incident light is assumed. Parameters of the DCF-based metamaterial: the same as in Fig. 3. EIT-based metamaterial is made of 20 identical layers shown in Fig. 2, with the parameters $L_2 = 4 \mu\text{m}$, $L_1 = 3.8 \mu\text{m}$, $w = t = 0.8 \mu\text{m}$, $g = 2.7 \mu\text{m}$, $h = 0.9 \mu\text{m}$, $s_y = 0.6 \mu\text{m}$, and periodicity $L_x = L_y = L_z = 7 \mu\text{m}$.

To underscore the advantages of the DCF-based approach to broadband SL, we compare a multilayer DCF-SL structure (unit cell is shown in Fig. 3) with its EIT-based counterpart (unit cell shown in Fig. 2, geometric parameters are given in the caption to Fig. 4). Both structures are based on silver antennas embedded in an $n = 1.5$ dielectric. Figure 4 (a) and (b) shows the COMSOL simulations results for DCF-SL and EIT-SL structures, respectively. The group velocity is calculated as $Ld\omega/d\phi$, where L is the total length of the structure and ϕ is the phase difference between incident and transmitted waves. In the case of EIT, the bandwidth can be potentially increased by increasing the coupling between dipole mode and quadrupole mode (κ_{xQ}). However, it

comes at the expense of the increased group velocity dispersion and transmission non-uniformity.

Specifically, both v_g and transmittance $|T|^2$ increase in the middle of the EIT band while remaining small near the EIT band edge. In contrast, the adiabatic DCF layers provide uniform group velocity as well as uniform cross-polarized transmittance inside the SL band. The bandwidth can be increased further simply by adding more adiabatically-varying layers. Note that the group velocity in the DCF-SL structure plotted in Fig. 4 is averaged over $N = 41$ layers. Inside the specific *resonant* layer, the group velocity of the corresponding frequency component is $< 0.01c$, which is consistent with the intensity enhancement of three orders of magnitude. The spectrally-flat transmission and absorption of the DCF-based metamaterial is appealing to a variety of linear and nonlinear applications. Moreover, because every frequency component is dramatically slowed down in a well-defined layer, applications to spectrally-selective active light manipulation can be envisioned.

In conclusion, we have proposed a new mechanism of slowing down light over a spectrally broad band by means of low-symmetry metamaterials exhibiting double-continuum Fano (DCF) resonance. This approach is conceptually different from the more common EIT-SL. It is shown that DCF-based broadband slow light with uniform group velocity and transmittance can be achieved by adiabatically changing the DCF resonance frequencies along the light propagation direction. Spectrally-flat light absorbers and filters, as well as various nonlinear devices requiring extreme light concentration are enabled by DCF-based slow-light metamaterials.

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